

Phys 460

Describing and Classifying Crystal Lattices

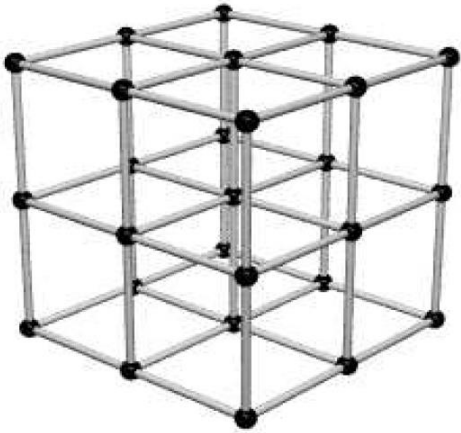
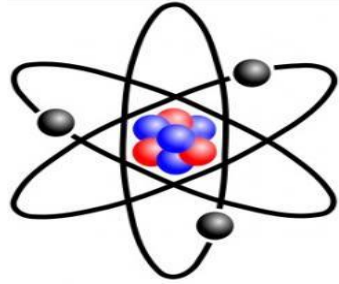
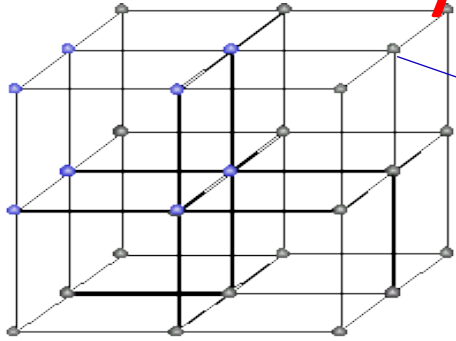


Figure 11.10: A simple cubic lattice

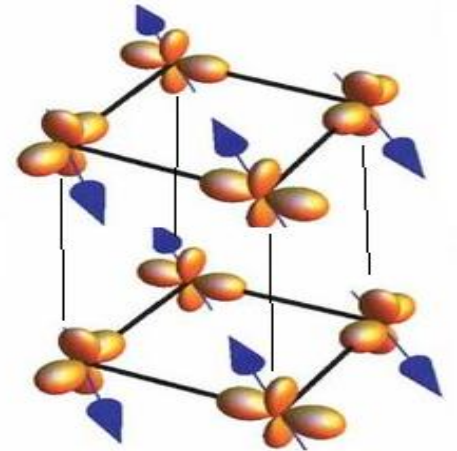
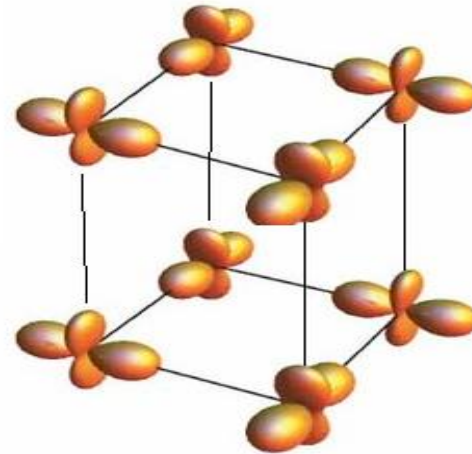
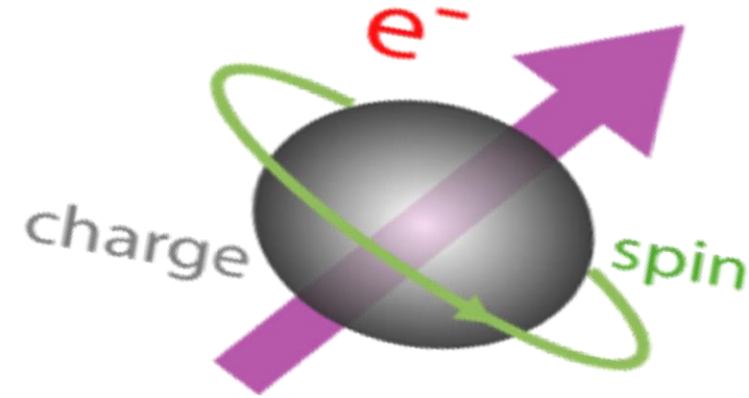


What is a “material”?

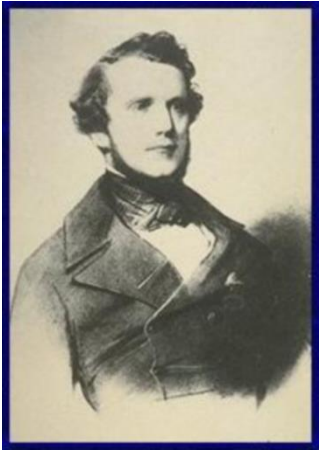
^{crystalline}



- Regular lattice of atoms
- Each atom has a positively charged nucleus surrounded by negative electrons
- Electrons are “spinning”
→they act like tiny bar magnets!
- Electrons don’t orbit like planets, but are distributed in space (“orbitals” or “Fermi sea”).
- Neighboring spins and orbitals talk to each other and form patterns!



Crystallography- Origins in geology



Auguste Bravais
1811-1863



- The atomic theory of atoms has long been proposed to explain sharp angles and flat planes (“facets”) of naturally occurring crystals.
- Types of different crystal structures were categorized, and formed the foundation of the field of *crystallography*.
- In 1848, French physicist (and crystallographer and applied mathematician) demonstrated that there were only 7 “types” of crystals, embodying 14 distinct “symmetries”.
- This work was verified and greatly expanded upon in the 20th century with the advent of *x-ray diffraction*. (e.g. work W. Henry and W. Lawrence Bragg, 1913)

Crystals- a mathematical description

- A *crystal* is defined to be a *repeating, regular array of atoms*.
- A proper mathematical description must account for *what is repeating* and *how it is repeating*.
- Two parts treated separately:
 - The fundamental repeating unit is referred to as the **basis**.
 - How the basis repeats is specified by identifying the underlying **lattice**.

Crystal structure = lattice + basis

(or “**Bravais lattice**” + basis)

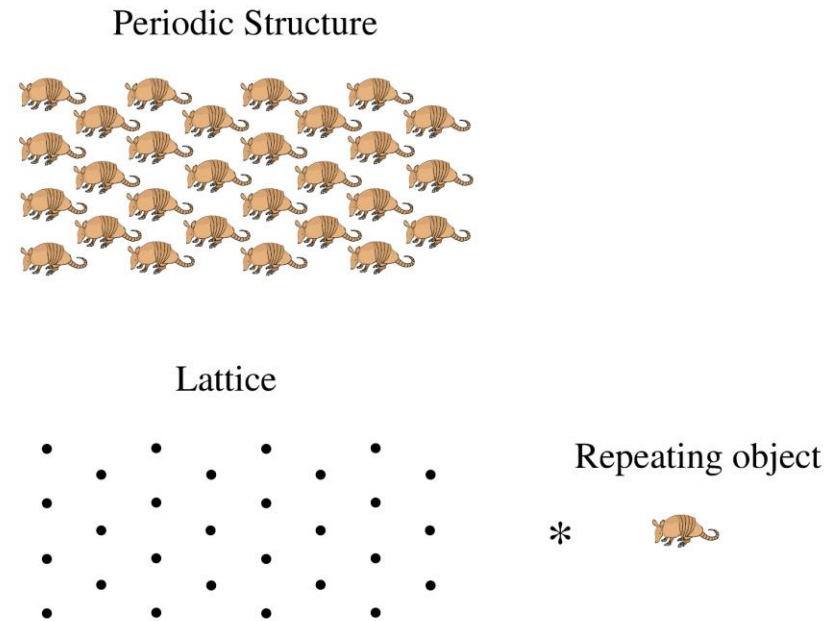


Figure 11.3: Any periodic structure can be represented as a lattice of repeating motifs.

Bravais lattice

- A **Bravais lattice** (what Simon simply calls a “lattice”) is a mathematical construct, designed to describe the underlying periodicity of a crystal.
- There are two *completely equivalent* definitions:
 1. A Bravais lattice is a set of all points in space with position vectors, \mathbf{R} , of the form
$$\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3$$
where \mathbf{a}_1 , \mathbf{a}_2 and \mathbf{a}_3 are any three independent vectors, and $n_i \in \mathbb{Z}$.
 2. A Bravais lattice is an *infinite* array of discrete points with an arrangement and orientation which looks *exactly* the same at each point.
- NB- Definition 2 is handy for first impressions, but definition 1 will form the foundation for our mathematical treatment.

Properties of a Bravais lattice

- Seemingly general, there are actually a finite set of possible Bravais lattices (14 in 3D), determined by the underlying symmetries (more on this later).
- Quick inspection shows that not every lattice is a Bravais lattice! A good counterexample is the common honeycomb:

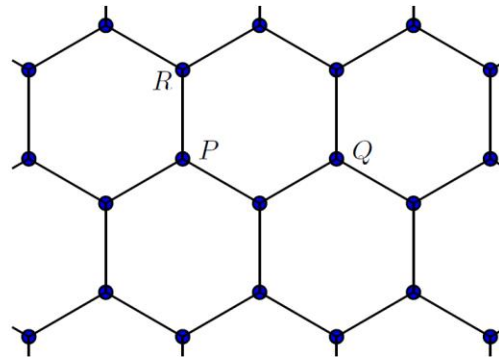


Figure 11.4: The honeycomb is not a lattice. Points P and R are inequivalent. (Points P and Q are equivalent)

Primitive vectors and unit cells

- If one is looking at a Bravais lattice, it is possible to describe all points using the position vector $\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3$. Here, the vectors \mathbf{a}_i are known as **primitive lattice vectors** and the integers n_i are known as **lattice indices**.
- In general, the repeating volume (area in 2D) in a crystal is known as the **unit cell**. For a Bravais lattice, the primitive lattice vectors span the smallest possible volume, and the resulting unit cell is called the **primitive unit cell**.
- Neither primitive basis vectors nor the primitive unit cells are unique!

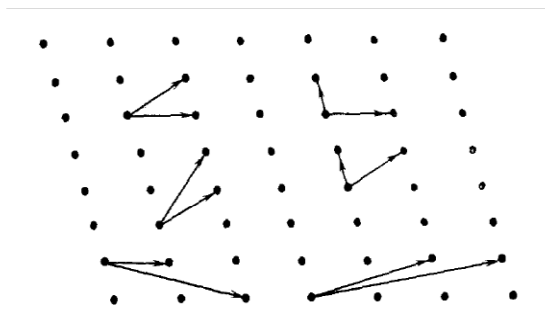


Figure 11.2: The choice of primitive lattice vectors for a lattice is not unique.

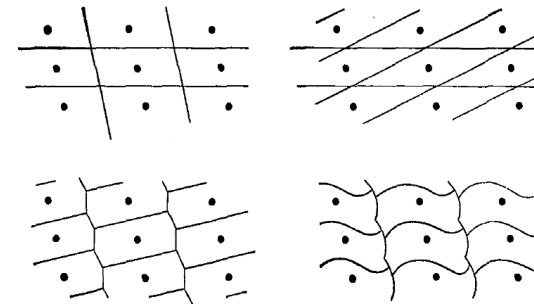


Figure 11.5: The choice of a unit cell is not unique. All of these unit cells reconstruct the same crystal.

Properties of primitive unit cells

- If every atom is described by $\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3$, then one can show that every *primitive cell* has *exactly one lattice point*.
- It follows that every primitive cell has exactly the same volume: $v = 1/n$, where n is point density.
- One can always find a primitive cell by considering parallelepiped spanned by primitive basis vectors.
- Another common choice is the **Wigner-Seitz** cell, created by *bisecting the lines connecting a lattice point with its nearest neighbors*.
 - Most symmetric cell, and volume closest to lattice point

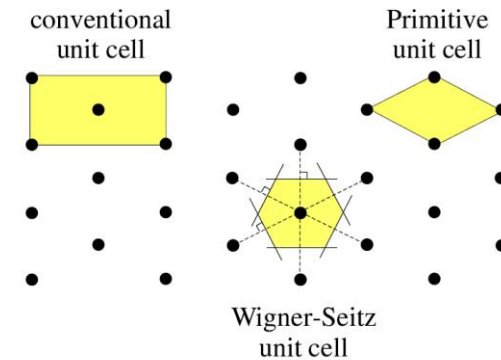


Figure 11.6: Some unit cells for the triangular lattice.

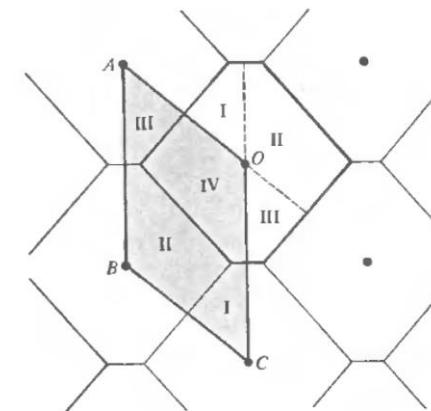
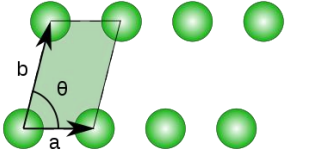
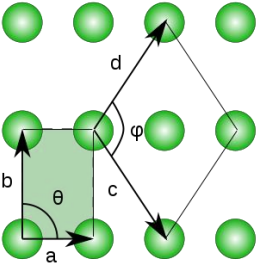
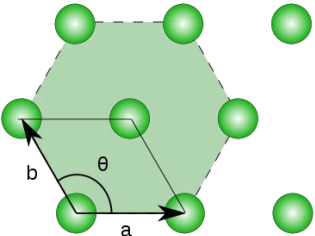
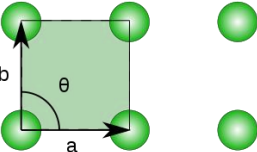
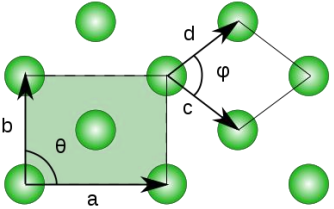


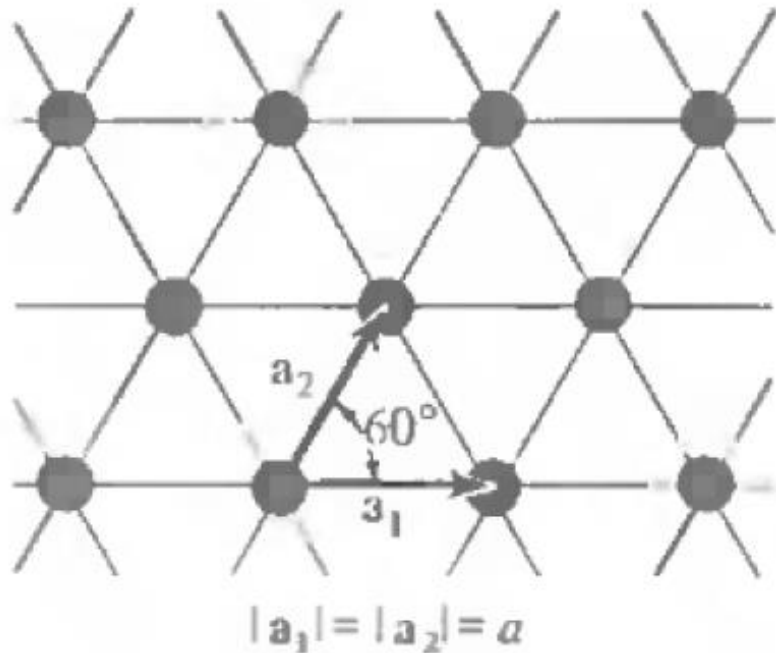
Figure 4.11

Two possible primitive cells for a two-dimensional Bravais lattice. The parallelogram cell (shaded) is obviously primitive; additional hexagonal cells are indicated to demonstrate that the hexagonal cell is also primitive. The parallelogram can be cut into pieces, which, when translated through lattice vectors, reassemble to form the hexagon. The translations for the four regions of the parallelogram are: Region I— \vec{CO} ; Region II— \vec{BO} ; Region III— \vec{AO} ; Region IV—no translation.

Bravais lattices in 2D

oblique	rectangular	hexagonal	cubic
 1	 2	 4	 5
	 3		
	centered rectangular		
$ a \neq b , \theta \neq 90^\circ$	$ a \neq b , \theta = 90^\circ$ $ c = d , \phi \neq 90^\circ$	$ a = b , \theta = 120^\circ$	$ a = b , \theta = 90^\circ$
m	o	h	t

Example: the triangle lattice (a.k.a. the hexagonal lattice)

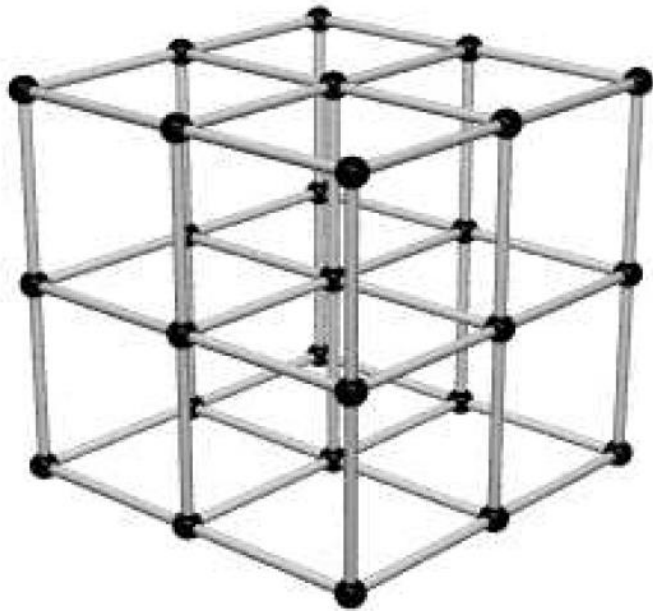


$$\mathbf{R} = n_1 \hat{\mathbf{a}}_1 + n_2 \hat{\mathbf{a}}_2$$

$$\mathbf{a}_1 = a \hat{\mathbf{x}}$$

$$\mathbf{a}_2 = \frac{a}{2} \hat{\mathbf{x}} + \frac{\sqrt{3}a}{2} \hat{\mathbf{y}}$$

3D Example: the simple cubic lattice



$$\mathbf{R} = n_1 \hat{\mathbf{a}}_1 + n_2 \hat{\mathbf{a}}_2 + n_3 \hat{\mathbf{a}}_3$$

$$\mathbf{a}_1 = a \hat{\mathbf{x}}$$

$$\mathbf{a}_2 = a \hat{\mathbf{y}}$$

$$\mathbf{a}_3 = a \hat{\mathbf{z}}$$

Figure 11.10: A simple cubic lattice

3D Example: the body-centered cubic (BCC)

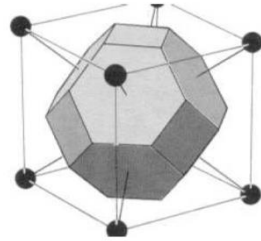
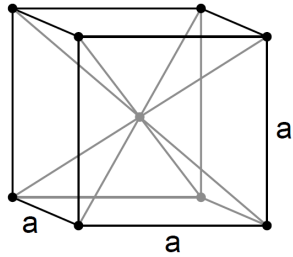
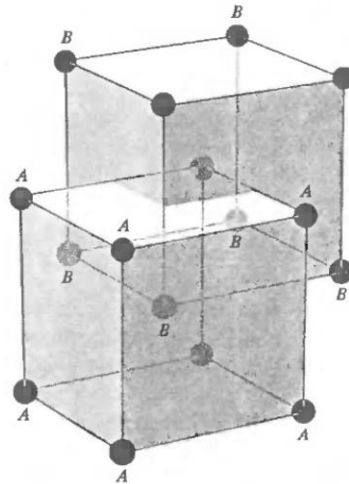


Figure 11.12: Conventional unit cell for the body centered cubic (I) lattice. Left: 3D view. Right: A plan view of the conventional unit cell. Unlabeled points are both at heights 0 and a.

Figure 4.5

A few sites from a body-centered cubic Bravais lattice. Note that it can be regarded either as a simple cubic lattice formed from the points *A* with the points *B* at the cube centers, or as a simple cubic lattice formed from the points *B* with the points *A* at the cube centers. This observation establishes that it is indeed a Bravais lattice.



$$\mathbf{R} = n_1 \hat{\mathbf{a}}_1 + n_2 \hat{\mathbf{a}}_2 + n_3 \hat{\mathbf{a}}_3$$

$$\mathbf{a}_1 = a \hat{\mathbf{x}}$$

$$\mathbf{a}_2 = a \hat{\mathbf{y}}$$

$$\mathbf{a}_3 = \frac{a}{2} (\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

OR

$$\mathbf{a}_1 = \frac{a}{2} (-\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_2 = \frac{a}{2} (\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_3 = \frac{a}{2} (\hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}})$$

3D Example: the face-centered cubic (FCC)

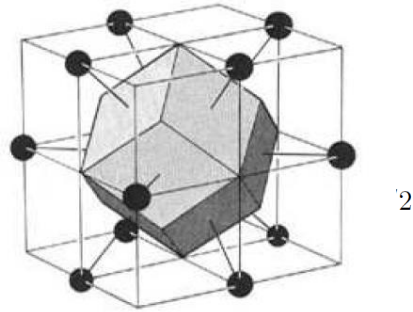
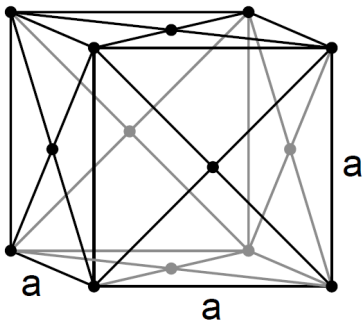
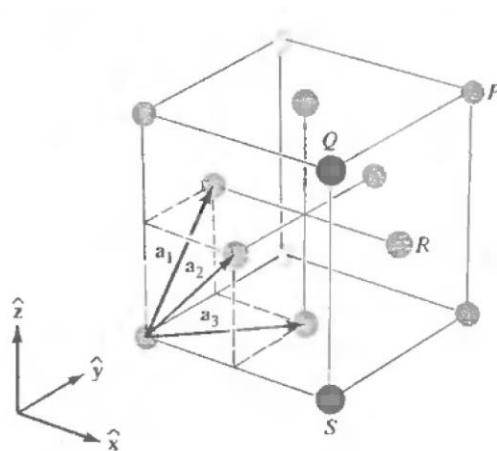


Figure 11.14: Conventional unit cell for the face centered cubic (F) lattice. Left: 3D view. Right: A plan view of the conventional unit cell. Unlabeled points are both at heights 0 and a .

Figure 4.9
A set of primitive vectors, as given in Eq. (4.5), for the face-centered cubic Bravais lattice. The labeled points are $P = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$, $Q = 2\mathbf{a}_2$, $R = \mathbf{a}_2 + \mathbf{a}_3$, and $S = -\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$.



$$\mathbf{R} = n_1 \hat{\mathbf{a}}_1 + n_2 \hat{\mathbf{a}}_2 + n_3 \hat{\mathbf{a}}_3$$

$$\mathbf{a}_1 = \frac{a}{2} (\hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_2 = \frac{a}{2} (\hat{\mathbf{x}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_3 = \frac{a}{2} (\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

Table 4.1
ELEMENTS WITH THE MONATOMIC FACE-CENTERED
CUBIC CRYSTAL STRUCTURE

ELEMENT	a (Å)	ELEMENT	a (Å)	ELEMENT	a (Å)
Ar	5.26 (4.2 K)	Ir	3.84	Pt	3.92
Ag	4.09	Kr	5.72 (58 K)	δ -Pu	4.64
Al	4.05	La	5.30	Rh	3.80
Au	4.08	Ne	4.43 (4.2 K)	Sc	4.54
Ca	5.58	Ni	3.52	Sr	6.08
Ce	5.16	Pb	4.95	Th	5.08
β -Co	3.55	Pd	3.89	Xe (58 K)	6.20
Cu	3.61	Pr	5.16	Yb	5.49

Data in Tables 4.1 to 4.7 are from R. W. G. Wyckoff, *Crystal Structures*, 2nd ed., Interscience, New York, 1963. In most cases, the data are taken at about room temperature and normal atmospheric pressure. For elements that exist in many forms the stable room temperature form (or forms) is given. For more detailed information, more precise lattice constants, and references, the Wyckoff work should be consulted.

Table 4.2
ELEMENTS WITH THE MONATOMIC BODY-CENTERED
CUBIC CRYSTAL STRUCTURE

ELEMENT	a (Å)	ELEMENT	a (Å)	ELEMENT	a (Å)
Ba	5.02	Li	3.49 (78 K)	Ta	3.31
Cr	2.88	Mo	3.15	Tl	3.88
Cs	6.05 (78 K)	Na	4.23 (5 K)	V	3.02
Fe	2.87	Nb	3.30	W	3.16
K	5.23 (5 K)	Rb	5.59 (5 K)		

Lattice with a basis

- A Bravais lattice is a mathematical abstraction. To describe a real **crystal lattice**, one needs to dress the Bravais lattice with either a single atom, or a set of atoms. This repeating set of atoms is called the **basis**.
- The vectors describing the positions of atoms in a basis, $\{\mathbf{r}_i\}$, are called **basis vectors**, and are conventionally presented in terms of fractional steps along the lattice vectors.
- This allows one to
 - Use the same Bravais lattice framework to Describe non-Bravais lattices
 - To simplify discussion of complex Bravais lattice through use of larger **conventional unit cells**, with orthonormal lattice vectors
 - To describe crystals containing more than one atom type.

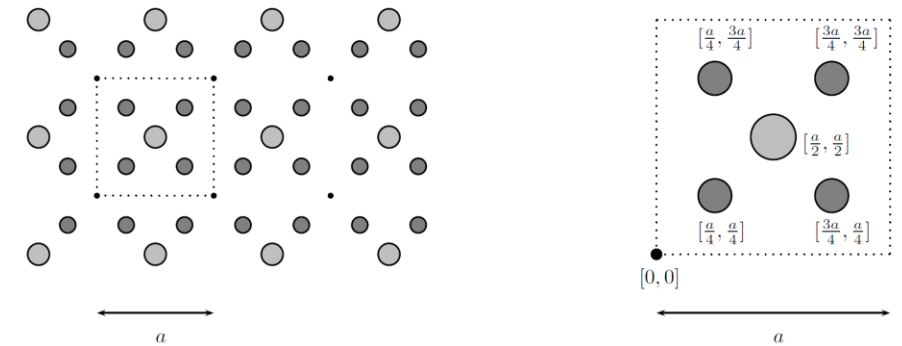


Figure 11.8: Left: A periodic structure in two dimensions. A unit cell is marked with the dotted lines. Right: A blow-up of the unit cell with the coordinates of the particles in the unit cell with respect to the reference point in the lower left hand corner. The basis is the description of the atoms along with these positions.

Example: The honeycomb, using a basis

- Though the honeycomb is clearly not a Bravais lattice, we can still describe it as a triangle lattice (which is Bravais), using a two atom basis

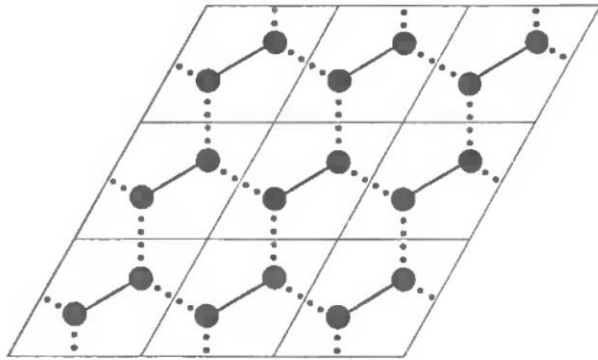


Figure 4.17
The honeycomb net, drawn so as to emphasize that it is a Bravais lattice with a two-point basis. The pairs of points joined by heavy solid lines are identically placed in the primitive cells (parallelograms) of the underlying Bravais lattice.

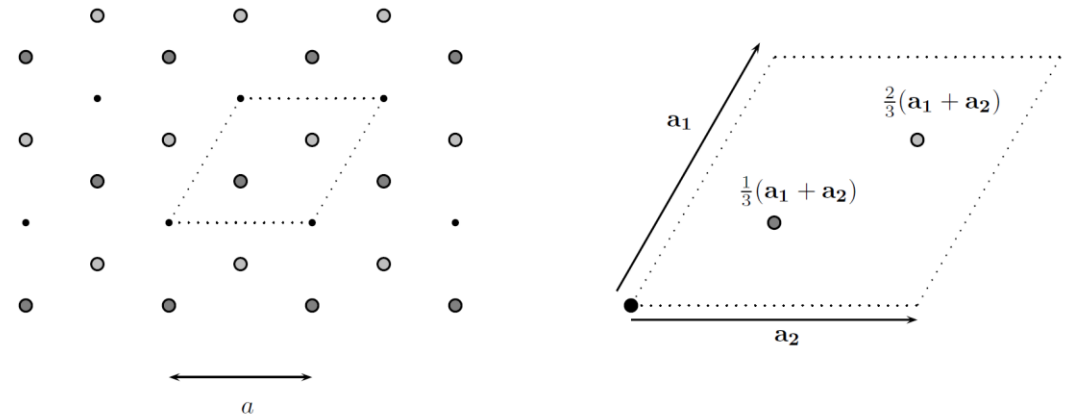
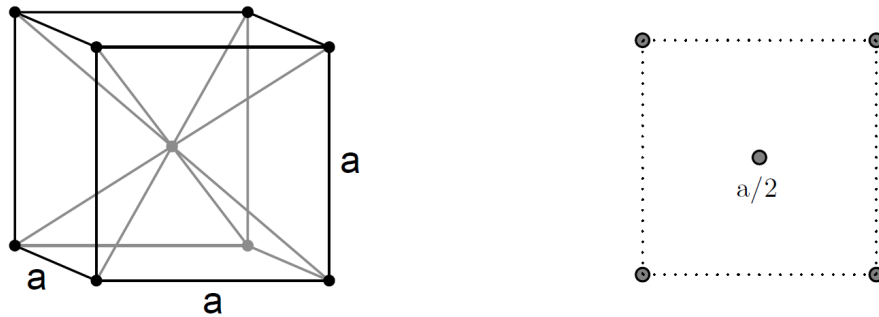


Figure 11.9: The honeycomb from Fig. 11.4 with the two inequivalent points of the unit cell given different shades. The unit cell is outlined dotted on the left and the corners of the unit cell are marked with small black dots. On the right the unit cell is expanded and coordinates are given with respect to the reference point written.

Example: the BCC lattice, using a basis

- The BCC lattice is Bravais, but the primitive lattice vectors are not perpendicular. It is often desirable to use a cubic Bravais lattice, with a two atom basis. The repeating (non-primitive) unit cell is known as the **conventional unit cell**.



Bravais lattice:

$$\mathbf{R} = n_1 \hat{\mathbf{a}}_1 + n_2 \hat{\mathbf{a}}_2 + n_3 \hat{\mathbf{a}}_3$$

$$\mathbf{a}_1 = a\hat{\mathbf{x}} \quad \mathbf{a}_2 = a\hat{\mathbf{y}} \quad \mathbf{a}_3 = a\hat{\mathbf{z}}$$

Basis:

$$\mathbf{r}_1 = \mathbf{0}$$

$$\mathbf{r}_2 = \frac{1}{2}(\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3)$$

Figure 11.12: Conventional unit cell for the body centered cubic (I) lattice. Left: 3D view. Right: A plan view of the conventional unit cell. Unlabeled points are both at heights 0 and a .

Example: the FCC lattice, using a basis

- The same conventional unit cell can be used to describe the FCC lattice, now with a 4 atom basis:

Bravais lattice:

$$\mathbf{R} = n_1 \hat{\mathbf{a}}_1 + n_2 \hat{\mathbf{a}}_2 + n_3 \hat{\mathbf{a}}_3$$

$$\mathbf{a}_1 = a\hat{\mathbf{x}} \quad \mathbf{a}_2 = a\hat{\mathbf{y}} \quad \mathbf{a}_3 = a\hat{\mathbf{z}}$$

Basis: $\mathbf{r}_1 = \mathbf{0}$

$$\mathbf{r}_2 = \frac{1}{2}(\mathbf{a}_1 + \mathbf{a}_2)$$

$$\mathbf{r}_3 = \frac{1}{2}(\mathbf{a}_1 + \mathbf{a}_3)$$

$$\mathbf{r}_4 = \frac{1}{2}(\mathbf{a}_2 + \mathbf{a}_3)$$

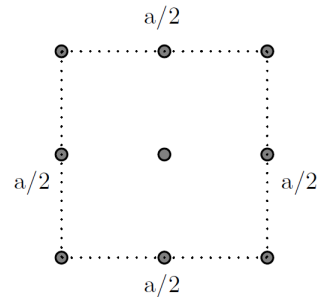
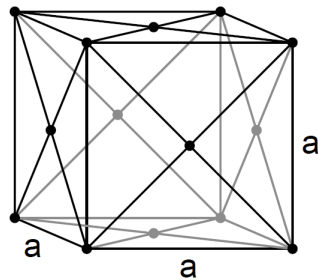


Figure 11.14: Conventional unit cell for the face centered cubic (F) lattice. Left: 3D view. Right: A plan view of the conventional unit cell. Unlabeled points are both at heights 0 and a .

Example: The diamond lattice

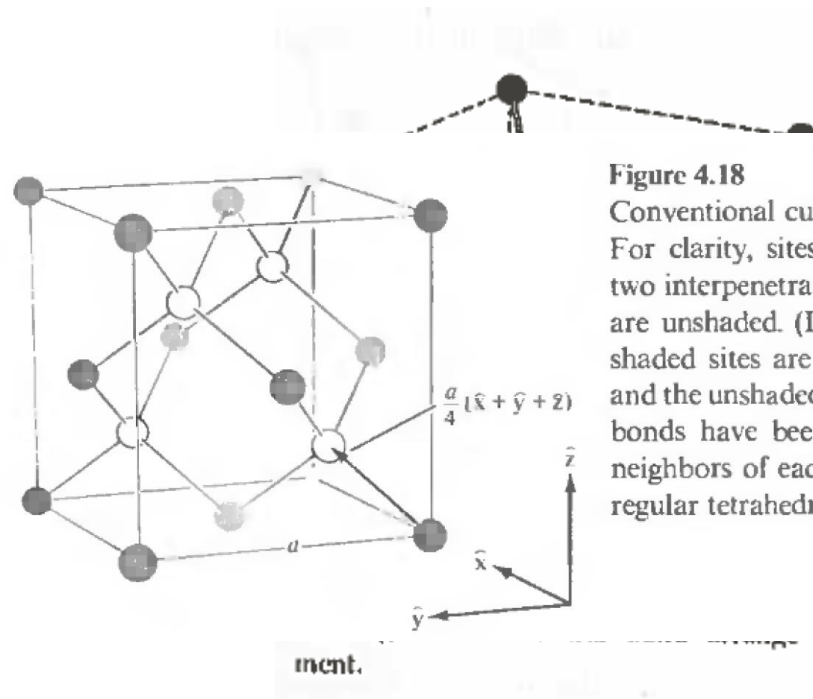


Figure 4.18

Conventional cubic cell of the diamond lattice. For clarity, sites corresponding to one of the two interpenetrating face-centered cubic lattices are unshaded. (In the zincblende structure the shaded sites are occupied by one kind of ion, and the unshaded by another.) Nearest-neighbor bonds have been drawn in. The four nearest neighbors of each point form the vertices of a regular tetrahedron.

Described as FCC lattice, with a 2-atom basis:

$$\mathbf{r}_1 = \mathbf{0}$$

$$\mathbf{r}_2 = \frac{a}{4}(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

where a is the length of the conventional cubic cell.

Note: It is equally well described as an FCC with an 8 atom basis (FCC basis plus the same vectors offset by $[a/4, a/4, a/4]$).

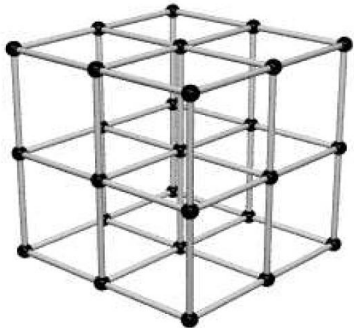
Table 4.3
ELEMENTS WITH THE DIAMOND CRYSTAL
STRUCTURE

ELEMENT	CUBE SIDE a (Å)
C (diamond)	3.57
Si	5.43
Ge	5.66
α -Sn (grey)	6.49

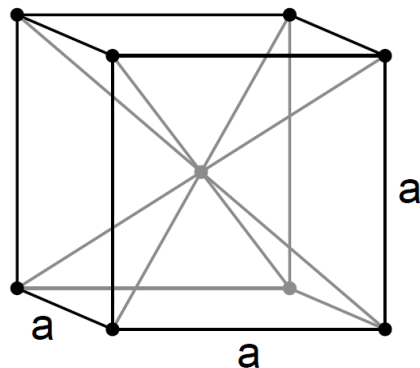
Sidebar: Coordination number

- Each of the above three lattices is incredibly common, and characterize different classes of elements. What is the difference?
- In many cases, the important difference is the number of nearest neighbors, which dictates how many *bonds* each atom has.
- The number of nearest neighbor bonds for a given crystal lattice is called the **coordination number**, and usually denoted Z .

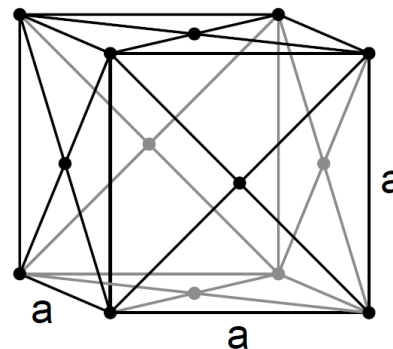
cubic, $Z=6$



BCC, $Z=8$



FCC, $Z=12$



diamond, $Z=4$

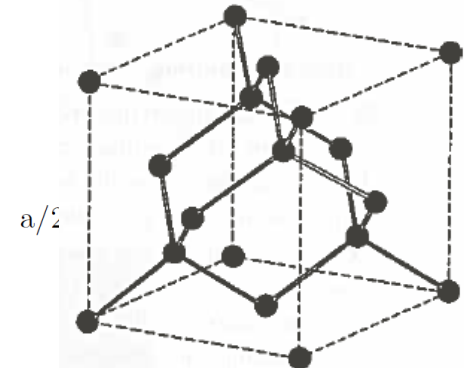


Figure 25 Crystal structure of diamond

Example: hexagonal close packed (hcp)

- Simple hexagonal lattice, with a 2 atom basis
- Represents closest approach of a series of hard spheres.

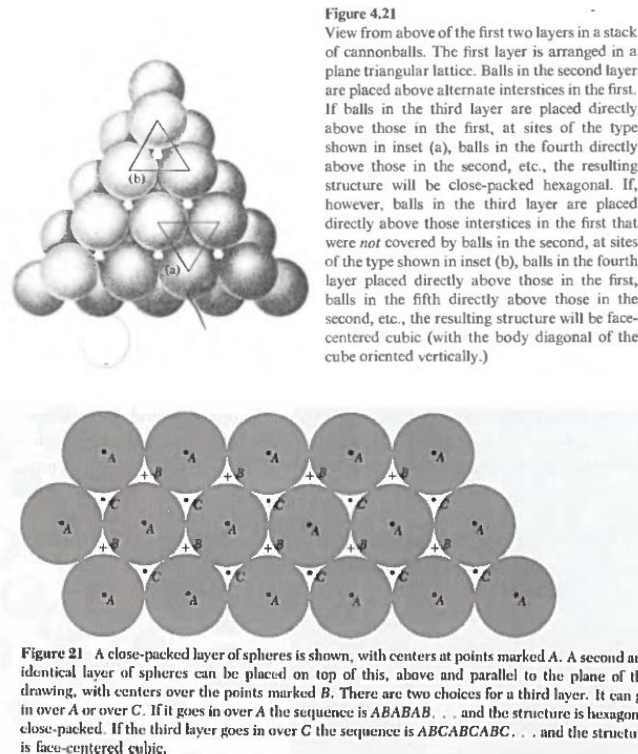
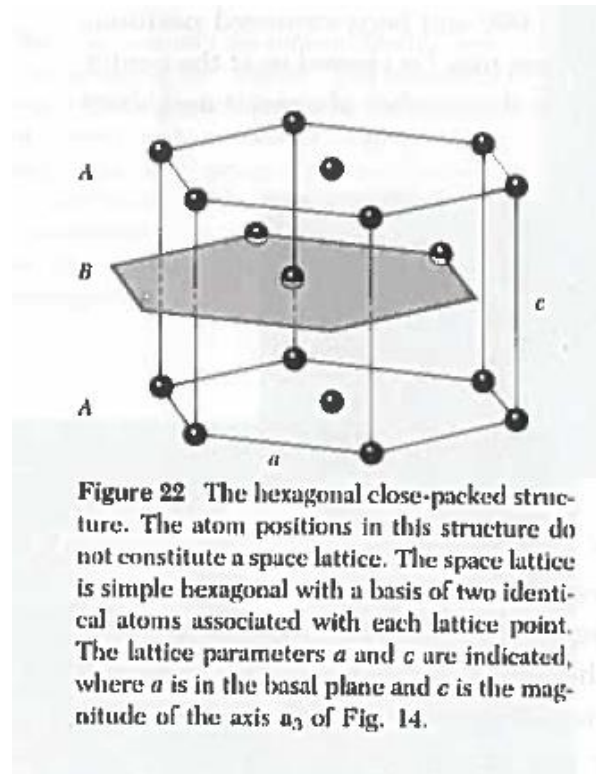


Figure 4.21

View from above of the first two layers in a stack of cannonballs. The first layer is arranged in a plane triangular lattice. Balls in the second layer are placed above alternate interstices in the first. If balls in the third layer are placed directly above those in the first, at sites of the type shown in inset (a), balls in the fourth directly above those in the second, etc., the resulting structure will be close-packed hexagonal. If, however, balls in the third layer are placed directly above those interstices in the first that were *not* covered by balls in the second, at sites of the type shown in inset (b), balls in the fourth layer placed directly above those in the first, balls in the fifth directly above those in the second, etc., the resulting structure will be face-centered cubic (with the body diagonal of the cube oriented vertically.)

Table 4.4

ELEMENTS WITH THE HEXAGONAL CLOSE-PACKED CRYSTAL STRUCTURE

ELEMENT	a (Å)	c	c/a	ELEMENT	a (Å)	c	c/a
Be	2.29	3.58	1.56	Os	2.74	4.32	1.58
Cd	2.98	5.62	1.89	Pr	3.67	5.92	1.61
Ce	3.65	5.96	1.63	Re	2.76	4.46	1.62
α -Co	2.51	4.07	1.62	Ru	2.70	4.28	1.59
Dy	3.59	5.65	1.57	Sc	3.31	5.27	1.59
Er	3.56	5.59	1.57	Tb	3.60	5.69	1.58
Gd	3.64	5.78	1.59	Ti	2.95	4.69	1.59
He (2 K)	3.57	5.83	1.63	Tl	3.46	5.53	1.60
Hf	3.20	5.06	1.58	Tm	3.54	5.55	1.57
Ho	3.58	5.62	1.57	Y	3.65	5.73	1.57
La	3.75	6.07	1.62	Zn	2.66	4.95	1.86
Lu	3.50	5.55	1.59	Zr	3.23	5.15	1.59
Mg	3.21	5.21	1.62				
Nd	3.66	5.90	1.61	"Ideal"			1.63

Common motifs in diatomic materials

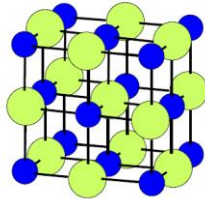
sodium chloride (NaCl)

lattice: cubic F

basis :

Na 000

Cl $\frac{1}{2}\frac{1}{2}\frac{1}{2}$



Plan view

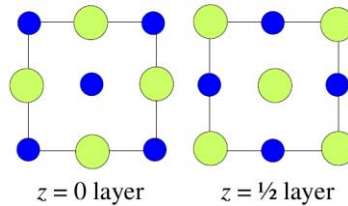


Table 4.5

SOME COMPOUNDS WITH THE SODIUM CHLORIDE STRUCTURE

CRYSTAL	a (Å)	CRYSTAL	a (Å)	CRYSTAL	a (Å)
LiF	4.02	RbF	5.64	CaS	5.69
LiCl	5.13	RbCl	6.58	CaSe	5.91
LiBr	5.50	RbBr	6.85	CaTe	6.34
LiI	6.00	RbI	7.34	SrO	5.16
NaF	4.62	CsF	6.01	SrS	6.02
NaCl	5.64	AgF	4.92	SrSe	6.23
NaBr	5.97	AgCl	5.55	SrTe	6.47
NaI	6.47	AgBr	5.77	BaO	5.52
KF	5.35	MgO	4.21	BaS	6.39
KCl	6.29	MgS	5.20	BaSe	6.60
KBr	6.60	MgSe	5.45	BaTe	6.99
KI	7.07	CaO	4.81		

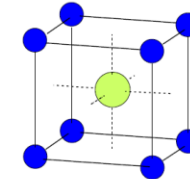
cesium chloride (CsCl)

lattice: cubic P

basis :

Cs 000

Cl $\frac{1}{2}\frac{1}{2}\frac{1}{2}$



Plan view

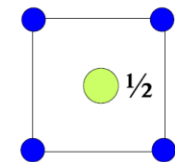


Table 4.6

SOME COMPOUNDS WITH THE CESIUM CHLORIDE STRUCTURE

CRYSTAL	a (Å)	CRYSTAL	a (Å)
CsCl	4.12	TlCl	3.83
CsBr	4.29	TlBr	3.97
CsI	4.57	TlI	4.20

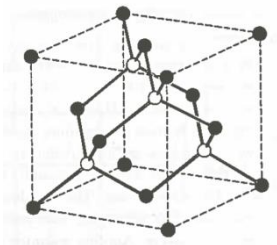
Zinceblende (ZnS)

lattice: cubic F

basis :

Zn 000

S $\frac{1}{4}\frac{1}{4}\frac{1}{4}$



Plan view

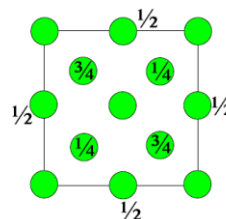


Table 4.7

SOME COMPOUNDS WITH THE ZINCBLLENDE STRUCTURE

CRYSTAL	a (Å)	CRYSTAL	a (Å)	CRYSTAL	a (Å)
CuF	4.26	ZnS	5.41	AlSb	6.13
CuCl	5.41	ZnSe	5.67	GaP	5.45
CuBr	5.69	ZnTe	6.09	GaAs	5.65
CuI	6.04	CdS	5.82	GaSb	6.12
AgI	6.47	CdTe	6.48	InP	5.87
BeS	4.85	HgS	5.85	InAs	6.04
BeSe	5.07	HgSe	6.08	InSb	6.48
BeTe	5.54	HgTe	6.43	SiC	4.35
MnS (red)	5.60	AlP	5.45		
MnSe	5.82	AlAs	5.62		

Sidebar: Indexing cubic directions and lattice planes

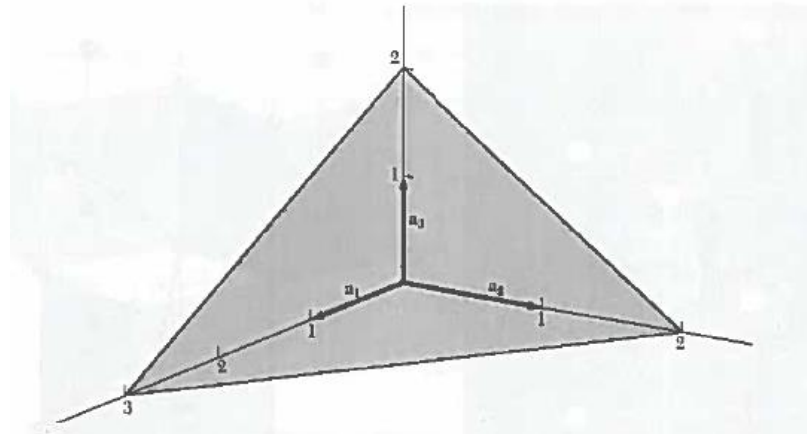


Figure 15 This plane intercepts the a_1 , a_2 , a_3 axes at $3a_1$, $2a_2$, $2a_3$. The reciprocals of these numbers are $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{2}$. The smallest three integers having the same ratio are 2, 3, 3, and thus the indices of the plane are (233).

Steps:

1. Find intercepts of plane with primary axes .
2. Take inverse of intercepts.
3. Multiply by lowest number to make all values integers

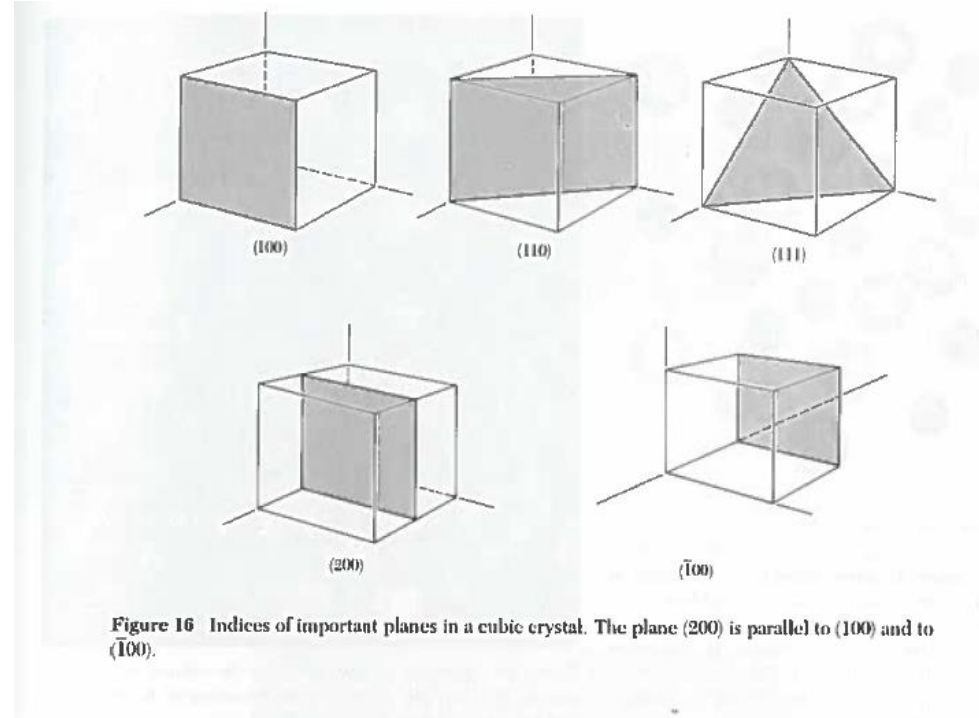


Figure 16 Indices of important planes in a cubic crystal. The plane (200) is parallel to (100) and to $\bar{1}00$.

Naming conventions:

1. (h,k,l) is the plane determined by procedure to the left
2. $[h,k,l]$ is the vector normal to this plane.
3. $\{h,k,l\}$ is the set of all symmetrically equivalent (h,k,l)

How do we classify lattices?

- One of the successes of crystallography (and why we remember the name Bravais) is that it manages to sort the infinite number of potential lattices in nature into a finite number categories.
- What category a lattice is in depends on the group of symmetry operators which can be applied which transforms a discrete set of coordinates (i.e. lattice points) into itself.
- The above examples dealt primarily with **translational symmetry operators**, and one way of thinking about primary lattice vectors is the direction and distance one can shift every atom to recreate the sample lattice.
- Additionally, there are a series of **point symmetry operators** that can be applied with an atom at the origin, which also bring the lattice into itself.
- The collection of point symmetries obeyed by a crystal define its **point group symmetries**. If one includes translation operators and all compound operations, this collection of symmetries defines the **space group** of the lattice.

Point group symmetries

- The point group symmetries are entirely captured by considering:
 - Rotations (2-,3-,4- and 6-fold) about any axis
 - Reflections across any plane
 - Inversion ($\mathbf{r} \rightarrow -\mathbf{r}$) for all lattice points
- For Bravais lattices (basis can't break symmetry), it turns out there are only 7 distinct point groups: cubic, tetragonal, orthorhombic, monoclinic, triclinic, trigonal and hexagonal.
- One can “derive” them by thinking about different ways of distorting a cube.

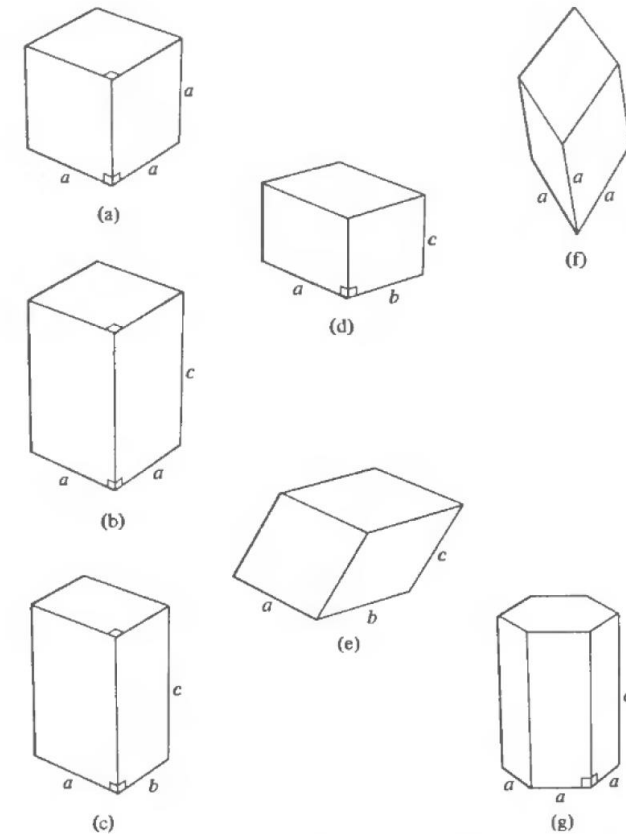


Figure 7.3
Objects whose symmetries are the point-group symmetries of Bravais lattices belonging to the seven crystal systems: (a) cubic; (b) tetragonal; (c) orthorhombic; (d) monoclinic; (e) triclinic; (f) trigonal; (g) hexagonal.

Table 7.4
ENUMERATION OF SOME SIMPLE SPACE GROUPS

SYSTEM	NUMBER OF POINT GROUPS	NUMBER OF BRAVAIS LATTICES	PRODUCT
Cubic	5	3	15
Tetragonal	7	2	14
Orthorhombic	3	4	12
Monoclinic	3	2	6
Triclinic	2	1	2
Hexagonal	7	1	7
Trigonal	5	1	5
Totals	32	14	61

Space groups of Bravais lattices

- Adding translation operators brings the total number of distinct symmetry groups to 14 for Bravais lattices in 3D
- Relaxing spherical symmetry of the basis brings the number of groups to the much larger (but still finite) number of 230.
- It is still common today amongst material physicist to refer a lattice's point group symmetry as a starting point for discussion → **these are worth knowing!**

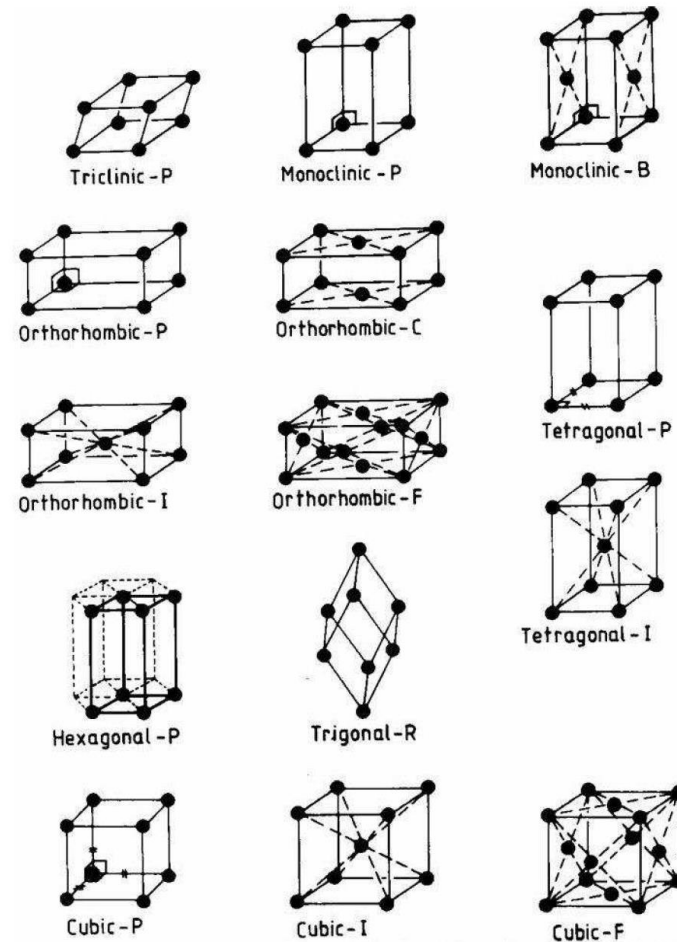


Figure 11.15: Unit cells for All of the Three Dimensional Bravais Lattice Types.

What's next?

- Now that we have a mathematical handle on how to describe infinite lattices of atoms (and the associated jargon), we can move forward to discuss how this periodicity affects material properties. In the coming weeks, we will discuss:
 1. Interaction of the lattice with light
 2. Collective motion of atoms, and its effect on heat capacity
 3. Motion of electrons in a periodic potential
 4. Lattices of spins and magnetic order
- For many (most) of these applications, it is more appropriate to talk about the discrete Fourier transform of the periodic lattice
 - i.e. we will use the **reciprocal lattice**, which will be the topic of next lecture